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OSCILLATIONS OF PLATES UNDER THE ACTION OF CONCENTRATED LOADS IN AN ACOUSTIC MEDIUM*

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Steady-state oscillations induced by concentrated forces acting on infinite isotropic and orthotropic plates in contact with a liquid are considered. On the basis of an asymptotic integration of the equations, an approximation method for solving the problem is proposed. Explicit expressions are obtained for the first approximations of deflections of the plates and the acoustic pressure in the medium. In the case of an isotropic plate, numerical integration of the equations in an exact formulation is also carried out and a comparison with the approximate analytic solution given. The isotropic plate problem has been previously considered /1—4/.Our study complements these results, and makes it possible to break up the initial problem into several simpler problems and to identify the structure of the integrals and simplify the solution method. As a result we are able to generalize our method to boundary value problems. A solution of the orthotropic plate problem (which has not previously been considered) is obtained.

1. Steady-state oscillations of an infinite isotropic plate in contact with a liquid and affected by a specified harmonic load $q(x, y)e^{-i\omega t}$ may be described by the following system of equations (the time factor is omitted everywhere below):

$$h_*^2\Delta_2^2 w - \lambda^2 w = (2Eh)^{-1}(q - P|_{z=0}), \quad \Delta P + k^2 P = 0 \tag{1.1}$$
$$\left.\frac{\partial P}{\partial z}\right|_{z=0} = \omega^2\rho w, \quad h_*^2 = \frac{h^2}{3(1-\nu^2)}, \quad \lambda = \frac{\omega}{c_0}, \quad k = \frac{\omega}{c}, \quad c_0 = \left(\frac{E}{\rho_0}\right)^{1/2}$$

Here w and h are the deflection and half-width of the plate; E, ν, and ρ_0, Young's modulus, the Poisson coefficient, and the density of the material of the plate; ρ and c, density of liquid and speed of sound; P, acoustic pressure; Δ and Δ_2, Laplacian operators in space and on the surface of the plate; ω, oscillation frequency; x, y rectangular Cartesian coordinates in the middle plane of the plate; and z, normal to the plate directed into the liquid. The exciting load $q(x, y)$ constitutes either a linerly concentrated force $Q\delta(x - x_0)$ referred to a unit of length or a concentrated force $Q\delta(x - x_0, y - y_0)$, referred to a unit of area.

The system of equations (1.1) contains the coefficient h_*^2 in the case of higher derivatives. In plate and shell theory, its ratio to the square of the characteristic linear dimension is taken as a small parameter. The choice of the characteristic dimension is not important. For the sake of definiteness, the wavelength l at the coincidence frequency /2/, at which the phase velocity of the free deflection waves in the plate is equal to the speed of sound in the liquid, may be suggested. The introduction of the small dimensionless parameter $h_*/l = (1/2\pi)(c/c_0)$ in (1.1) is made by "stretching" the coordinates x and y by a factor of $l^{-1/2}$. Here the form of the equations (1.1) remains unchanged.

Because of the presence of the small parameter, it is possible to construct asymptotic integration processes, that is so-called first and second iterational processes, by means of which the slowly and rapidly varying components of the solution may be correspondingly determined. With reference to equations of shell theory, such processes have been previously constructed /5/ on the right side using a δ-function. In this case, where the equation of oscillation of the plate with a δ-function on the right side is complemented with the equation of the oscillation of the liquid, the problem must be stated somewhat differently. However, the basic approach to the construction of iterational processes is retained. We will construct a solution of the initial problem following this method, and then estimate the error of the solution.

Let us first consider the oscillation problem for a plate resting on a liquid and subjected to a linearly concentrated force applied along the line $x = x_0$. In this case, the problem is two-dimensional. In the first approximation, we represent the pressure of the liquid near the plate in the form /6/

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$$P(x, z) = P(x, 0)\, e^{-az} \tag{1.2}$$

(a is an unknown parameter). Where $|a|$ is high enough, this representation corresponds to rapidly varying boundary oscillations of the liquid.

We substitute expression (1.2) in equation (1.1). Using the relation between the normal derivative of pressure on the surface of the plate and the deflection, we express $P(x, 0)$ in equations (1.1) in terms of w. The system of equations (at $z = 0$)

$$h_*^2\Delta_2^2 w - \lambda^2(1 + a^{-1}g)\, w = (2Eh)^{-1}Q\delta(x - x_0), \quad g = \rho(2h\rho_0)^{-1} \tag{1.3}$$
$$\Delta_2 w + (a^2 + k^2)\, w = 0, \quad P(x, 0) = -\omega^2\rho a^{-1}w, \quad \Delta_2 = \partial^2/\partial x^2$$

is obtained, in which the compatibility condition is given by the equality

$$(a^2 + k^2)^2 = \Omega_0^4(1 + a^{-1}g), \quad \Omega_0^4 = \lambda^2 h_*^{-2} \tag{1.4}$$

which constitutes a fifth-degree algebraic equation in a.

From an analysis of equation (1.4), it is clear that one of its roots $a_1 \sim \Omega_0$, two of its complex-conjugate roots a_2, $a_3 = \bar{a}_2$ have positive real part, further $|a_2| \sim a_1$, $\operatorname{Re} a_2 \ll a_1$, and two of its complex-conjugate roots have negative real part. The roots $a_{1,2,3}$ correspond, by (1.2), to an acoustic pressure component that attenuates with distance from the plate, and the root with negative real part corresponds to the increase in the component. Below we will prove that the increasing pressure component exerts a weak influence on the plate's modulus of flexure. In constructing the first approximation of the solution, therefore, we will take into account only the roots $a_{1,2,3}$.

With each root $a_{1,2,3}$, we may associate definite integrals of equations (1.3). Integrals that oscillate about the x-axis correspond to the root a_1, while integrals that attenuate by an oscillating exponential law with distance from the line of action of the force correspond to the roots $a_{2,3}$. The general solutions (w_+, P_+ with $x \geqslant x_0$, w_-, P_- if $x \leqslant x_0$) satisfying the condition for diverging waves has the form

$$w_\pm = \sum_{j=1}^{3} c_j^\pm \exp[\mp s_{j-1}(x - x_0)], \quad s_0 = -i(a_1^2 + k^2)^{1/2}, \quad s_2 = \bar{s}_1 \tag{1.5}$$
$$P_\pm = -\omega^2\rho \sum_{j=1}^{3} c_j^\pm a_j^{-1} \exp[\mp s_{j-1}(x - x_0) - a_j z], \quad s_1 = i(a_2^2 + k^2)^{1/2}$$

The constants $c_j^\pm$ $(j = 1, 2, 3)$ are determined under the assumption that the solutions (1.5) are conjugate along the line of action of the force:

$$w_+^{(n)} - w_-^{(n)} = \delta_{n3}QD^{-1}, \quad P_+^{(m)}(x, 0) - P_-^{(m)}(x, 0) = 0, \quad D = 2Ehh_*^2$$

Once we have determined the constants $c_j^\pm$, we arrive at an expression for the deflection:

$$w(x) = \frac{Q}{2Dd_0}\left[i\,\frac{b_2}{b}\exp(ib|x - x_0|) + \operatorname{Im}\left(\frac{b_{13}}{s_1}\exp(-s_1|x - x_0|)\right)\right] \tag{1.6}$$
$$d_0 = \operatorname{Im}[b_{13}(a_1^2 - a_2^2)], \quad b_2 = \operatorname{Im} a_2^{-1}, \quad b = is_0, \quad b_{13} = a_1^{-1} - a_3^{-1}$$

This equation has two terms, the first of which corresponds to a diverging, undamped wave, while the second, to a standing wave that rapidly damped with distance from the line of action of the force.

Let us now consider the case of a concentrated force. A zero-order Hankel function of the first kind $H_0^{(1)}$ (in the case of a_1) and modified Hankel functions K_0 (in the case of $a_{2,3}$) which depend upon the polar radius r, are integrals of equation (1.3) corresponding to the roots a_j $(j = 1, 2, 3)$ and which satisfy the condition for diverging waves. The general solution.

$$w(r) = c_1 H_0^{(1)}(br) + c_2 K_0(s_1 r) + c_3 K_0(s_2 r) \tag{1.7}$$

has, in general, a logarithmic singularity at the point of application of the force. However, it is known that the selection of a plate in a vacuum has a principal singularity of the form $r_0^2 \ln r_0$ /5/, where r_0 is the dimensionless radius. By (1.3), the pressure singularity $P(x, 0)$ is of the same order of magnitude. Therefore, the coefficients of $\ln r_0$ in the expansion of the functions w and $P(x, 0)$ in a neighborhood of the point of application of the force must vanish, while the coefficient of the principal singularity w is a known value $\varkappa$ /7/.

These requirements lead to the system of equations

$$c_{10} + c_2 + c_3 = 0, \; s_0^2 c_{10} + s_1^2 c_2 + s_2^2 c_3 = -4\varkappa, \; c_{10} = -2i\pi^{-1}c_1 \tag{1.8}$$
$$a_1^{-1}c_{10} + a_2^{-1}c_2 + a_3^{-1}c_3 = 0, \quad \varkappa = (8\pi D)^{-1}Q$$

Substituting the constants c_j determined from these conditions in (1.7) and taking into account the relation between the deflection and pressure (1.3), we obtain an approximate solution of the problem in the form

$$w(r) = 4\varkappa d_0^{-1}\left[i(\pi/2)\,b_2 H_0^{(1)}(br) - \mathrm{Im}\,(b_{13}K_0(s_1, r))\right] \tag{1.9}$$
$$P(r, z) = -\omega^2\rho\,\frac{4\varkappa}{d_0}\left\{i\,\frac{\pi}{2}\,\frac{b_2}{a_1}\,H_0^{(1)}(br)\,e^{-a_1 z} - \mathrm{Im}\left[\frac{b_{13}}{a_2}\,K_0(s_1 r)\,e^{-a_2 z}\right]\right\}$$

Let us compare this solution with previously published solutions of the problem. We start with the asymptotic solution of (1.9) for the case of great distances between the point of application of the force (at $kr \gg 1$) and the asymptotic curve obtained by the saddle-point method /4/.

The asymptotic expression for $P(r, z)$ obtained from (1.9) in the case of $kr \gg 1$ may be written in the form

$$P(r, z) = -ib\lambda^2 Q/(4hh_*^2 a_1 d_0 \sqrt{2\pi br})\exp\left[i(br - \pi/4) - a_1 z\right] \tag{1.10}$$

The asymptotic curve of a surface pressure wave has the form /4/

$$P(r, z) = -\frac{ig^3}{ks^2}\,Q\sqrt{\frac{q_1^2 - 1}{2\pi krq_1}}\;\frac{\exp\left[i(krq_1 - \pi/4) - kz\sqrt{q_1^2 - 1}\right]}{5q_1^4 - 4q_1^2 - \beta^3} \tag{1.11}$$
$$q_1^5 - \beta^2 q_1 - s\beta^3 = 0,\quad s = gh_* c_0 c^{-1},\quad \beta = g\,(ks)^{-1}$$

These expressions for the values of the wave thickness $2kh$ under which $\beta \gg 1$, yield extremely close numerical results. When $\beta < 1$, formula (1.11) is not applicable, whereas no such constraint applies in the case of formula (1.11). This asymptotic formula becomes more precise with increasing kh.

The size of the plate deflection determined from (1.9) at the point of application of the force was compared by means of the formulas

$$w(0) = \mathrm{Re}\,w(0) + i\,\mathrm{Im}\,w(0),\quad \mathrm{Im}\,w(0) = 2\pi\varkappa b_2 d_0^{-1} \tag{1.12}$$
$$\mathrm{Re}\,w(0) = -4\varkappa d_0^{-1}\,\mathrm{Im}\,[b_{13}\ln(bs_1^{-1})]$$

to the deflection induced by a force as found in /1/. A comparison for the case of a plate in contact with water ($\rho_0/\rho = 7,8$) showed that the values of the modulus of deflection become closer together with the increasing wave half-width of the plate kh. The greatest divergence is found in the case of the minimal values $kh = 0.005$ found in the computations, and amounts to 10.6%.

Our solution (1.9) was also compared to a numerical solution of the initial problem (1.1) in the case of a concentrated force. The values P and w were computed using the formulas

$$w(r) = \frac{-Q}{4\pi h\rho_0 c^3}\int_i^{i+\infty} S(kr, \eta)\,\eta\,d\eta,\quad S(kr, \eta) = \frac{\eta J_0(kr\sqrt{1+\eta^2})}{\beta_1\eta(1+\eta^2)^2 - \eta - g/k} \tag{1.13}$$
$$P(r, z) = \frac{gk}{2\pi}\,Q\int_i^{i+\infty} S(kr, \eta)\,e^{-\eta kz}\,d\eta,\qquad \beta_1 = \left(kh_*\frac{c^\circ}{c}\right)^2$$

obtained by means of the Hankel transformations /1/. In (1.13), J_0 is a zero-order Bessel function of a complex argument.

Results of computations of the deflection at the point of application of a force are presented below, where w_R, w_I and w_R°, w_I° denote the real and imaginary parts of the deflection computed using formulas (1.12) and (1.13) and referred to $2h$ and multiplied by a factor of 10^6.

$2kh$	0.01	0.05	0.10	0.15	0.20
w_R	308	32	10	5	3
w_I	−4711	−1113	−586	−400	−304
w_{R°	884	182	93	61	44
w_{I°	−4252	−1041	−564	−395	−306

It is clear that the values of $|w_R|$ are much less than $|w_I|$. The imaginary part is also greater than the normal derivative of the pressure of the liquid on the plate surface.

The solid cruve in Fig.1 denotes the real and imaginary parts of the dimensionless pressure $P^* = P_R^* + iP_I^*$, $P^* = P/(kgQ)$ plotted as a function of z at $r = 0$ and $kh = 0.1$, as computed using formulas (1.13).

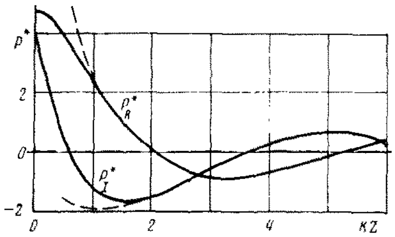

Fig.1

These computations show that the real part of the pressure on the plate surface and the real part of the deflection corresponding to it need not be taken into account in a first approximation when determining the modulus of deflection near the point of application of the force.

The imaginary part of the deflection determined using (1.12) are close to results of numerical computations. The divergence between them amounts of 10.8% at $2kh = 0.01$, and decreases with an increase in kh, amounting to 0.7% when $kh = 0.1$. In determining the real part of the deflection, the error in (1.12) is significant. This may be attributed to the fact that its magnitude is small, not exceeding the error of the first approximation of the solution of (1.12).

To correctly determine the real part of the deflection, it is necessary to take into account the slowly varying component of the solution. According to the asymptotic method of integration, the first approximation of this component may be determined from the system of equations (1.1) by setting the coefficient h_*^2 of the higher derivatives equal to zero. The equation of the plate oscillations and nonflowing conditition thereby reduced to the condition (at $z = 0$)

$$-g^{-1}\partial P/\partial z + P(x, y, 0) = Q\delta(x, y) \tag{1.14}$$

which turns the determination of the pressure P into a boundary-value problem, independent of the plate problem.

Thus, the asymptotic method of integration breaks up the initial problem of the compatible oscillations of a plate and liquid into two independent problems: (1) short-wave plate oscillations accompanied by boundary oscillations of the liquid, and (2) long-wave oscillations of the liquid.

The pressure field in the liquid was computed using formulas (1.13) in the case of both the complete equations of oscillations of the plate (1.1) as well as for the degenerate equations $(h_*^2 = 0)$. A comparison of the results of these computations confirm the correctness of the asymptotic representation of the solution of the Helmholtz equation with boundary condition (1.14) as the slowly varying components of the exact solution of the initial problem. The solid curves in Fig.1 show the real and imaginary parts of the pressure computed for the case of a degenerate boundary condition (1.14). Clearly, the divergence from the exact values of the corresponding components is substantial only near the plate. When $kz > 2$, it becomes negligibly small. Thus, the asymptotic approach may be used to compute the pressure in a liquid, beginning at some distance from the plate, independent of the computation of the type of oscillations of the plate.

The near pressure field may be computed using the asymptotic formula (1.9). However, it must be borne in mind that when $z \neq 0$, expression (1.9) for P has a logarithmic singularity with respect to r. The coefficient of $\ln r_0$ behaves with increasing z first as an increasing function, and then attenuates exponentially. Consequently, expression (1.9) for P coincides with the attenuating component of the rapidly varying part of the exact solution of problem (1.1) near the plate everywhere other than at points on the z-axis $(r = 0)$. If the expression for P from (1.9) is substituted in the Helmholtz equation (1.1), a discrepancy appears in the form of a product of the δ-function of r and the function of z, equal to the coefficient of $\ln r_0$. This discrepancy may be eliminated using the second approximation of the solution of problem (1.1).

Let us estimate the order of magnitude of the second approximation. For this purpose, we represent the solution of the initial equations (1.1) in the following form

$$P = P_0 + P_1, \ w = w_0 + w_1$$

where P_0 and w_0 denote the functions in (1.9). The equations for P_1 and w_1 obtained by substituting P and w in (1.1) have the form

$$\begin{gathered} h_*^2 \Delta^2_2 w_1 - \lambda^2 w_1 + (2Eh)^{-1} P_1(x, y, 0) = 0, \ \Delta P_1 + k^2 P_1 = \delta(x, y) F(z) \\ \left.\frac{\partial P_1}{\partial z}\right|_{z=0} = \omega^2 \rho w_1, \qquad F(z) = \omega^2 \rho \frac{4\varkappa}{d_0} \left[\frac{b_2}{a_1} e^{-a_1 z} - \operatorname{Im}\left(\frac{b_{13}}{a_2} e^{-a_2 z}\right)\right] \end{gathered} \tag{1.15}$$

The order of magnitude of w_1 with respect to w may be found by means of a Fourier transformation of the systems (1.1) and (1.15) in the plane of the plate. Integrating the ordinary differential equations in z relative to the mappings $p(s, z)$ and $p_1(s, z)$ and relating them to the mappings $w(s)$ and $w_1(s)$, then estimating the inverse transformations, retaining higher orders of the quantities, we arrive at the conclusion that, relative to w, w_1 constitutes (in the sense of a norm) a quantity on the order $\operatorname{Re} a_2/a_1 \ll 1$, where a_1 and a_2 are the roots of the algebraic equation (1.4). Hence, our solution w_0 is a good approximation for high enough values of a_1. This holds in the range of high and intermediate frequencies. It may be improved only at low oscillation frequencies $(2kh < 0.01)$. Note that this range of applicability of the first approximation in our method extends the domain of application of the solution obtained in /2,4/ and gives good results when $kh \ll 1$.

2. The method developed above can be generalized to oscillation problems in which the oscillations are induced by concentrated forces applied to anisotropic plates interacting with a liquid. Previously developed methods for the case of isotropic plates are not efficient in this case. Without any limits on generality, we will consider an orthotropic plate, in which the operator $D\Delta_2^2$ is replaced by

$$D_1 \frac{\partial^4}{\partial x^4} + 2D_3 \frac{\partial^4}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4}{\partial y^4}$$

where D_i $(i = 1, 2, 3)$ are the rigidity coefficients (see, for example /8/). In this case, equation (1.1) may be solved by means of the plane wave method /9/.

Let us represent the deflection of the plate, pressure in the liquid and δ-function in the form of integrals of the plane wave

$$w(x, y) = \int_0^{2\pi} W(\xi)\, d\beta, \quad P(x, y, z) = \int_0^{2\pi} \Phi(\xi, z)\, d\beta \tag{2.1}$$

$$\delta(x, y) = -\frac{1}{(2\pi)^2} \int_0^{2\pi} \frac{d\beta}{\xi^2(\beta)}, \quad \xi = x\cos\beta + y\sin\beta$$

Substituting these integrals in (1.1) and (1.2) and equating the integrands on the left and right sides, we obtain a system of equations for a plane wave. When $z = 0$, this system assumes the form

$$\gamma^2(\beta) \frac{d^4 W}{d\xi^4} - \Omega^4\left(1 + \frac{g}{a}\right) W = -\frac{Q}{(2\pi\xi)^2}, \quad \frac{d^2 W}{d\xi^2} + (a^2 + k^2) W = 0 \tag{2.2}$$

$$\gamma^2(\beta) = D_1 \cos^4\beta + 2D_3 \cos^2\beta \sin^2\beta + D_2 \sin^4\beta, \ \Omega^4 = 2h\rho_0\omega^2$$

which is analogous to equations (1.3). The corresponding algebraic equations for the pressure attenuation indicators a also coincides in form with (1.4) and since $\gamma^2(\beta) > 0$, remains an invariant classification of its roots. Therefore, the solution (1.6) obtained for the case of oscillations of an isotropic plate affected by a linearly concentrated force may be used as a Green's function to solve the nonhomogeneous system (2.2) in the oscillation problem for an orthotropic plate subjected to a concentrated force. If expression (1.6) is denoted by $w_0(x, x_0)$, the solution (2.2) is written in the form

$$W(\xi) = \frac{-1}{4\pi^2} \int_{-\infty}^{\infty} w_0(\xi, \xi_0) \frac{d\xi_0}{\xi_0^2} \tag{2.3}$$

The cylindrical rigidity of an isotropic plate D occurring in $w_0(x, x_0)$ is replaced here by $\gamma^2(\beta)$. The integrand $w_0(\xi, \xi_0)\xi_0^{-2}$ in (2.3) has an exponential singularity at the point $\xi_0 = 0$, at which it is locally nonintegrable. As has been previously proved /9/, the integral of such a function may be treated as a Cauchy principal value. Applying the general rule to (2.3) we arrive at an integral of the form

$$W(\xi) = -\frac{1}{4\pi^2} \int_{-\infty}^{\infty} \frac{\partial w_0}{\partial \xi_0} \frac{d\xi_0}{\xi_0} \tag{2.4}$$

with integrable singularity at zero.

Substituting expressions (1.6) in (2.4) and performing termwise integration, we arrive at an expression for $W(\xi)$ in terms of the integral functions

$$W = \frac{Q}{4\pi^2\gamma^2 d_0} \left[\frac{\pi}{2} i b_2 \cos x_1 - b_2 F_2 - \operatorname{Im}(b_{13} F_1)\right] \tag{2.5}$$

$$F_1 = E_1(z_1) \operatorname{ch} z_1 - e^{-z_1} \operatorname{Shi}(z_1), \quad F_2 = \operatorname{Ci}(x_1) \cos x_1 + \operatorname{Si}(x_1) \sin x_1$$

$$E_1(z_1) = \int_{z_1}^{\infty} \frac{e^{-t}}{t} dt, \quad \operatorname{Shi}(z_1) = \int_0^{z_1} \frac{\operatorname{sh} t}{t} dt, \quad \operatorname{Si}(x) = \int_0^{x} \frac{\sin t}{t} dt$$

$$\operatorname{Ci}(x) = -\int_x^{\infty} \frac{\cos t}{t} dt, \quad x_1 = b|\xi|, \quad z_1 = s_1|\xi|, \quad |\arg z_1| < \pi/2$$

If in (2.2) we set the rigidity coefficients $D_i = D\ (i = 1, 2, 3)$, we can prove that expression (2.5) may be transformed into (1.9) for the case of the deflection of an isotropic plate in a liquid oscillating as a result of the effect of a concentrated force.

By substutiting (2.5) in (2.1), we arrive at the required solution of the oscillation problem for an orthotropic plate in a liquid subjected to a concentrated force in the form of the quadrature of known functions. These functions cannot be integrated using standard tabular forms, and so integration is performed numerically.

As an example, we present results of a computation using (2.5) for the case of a fibre glass-reinforced plastic plate 1cm thick in contact with water and having the following elasticity parameters: $E_1 = 2.26 \cdot 10^6$ N/cm^2; $E_1/E_2 = 1.35$, $G = 0.346 \cdot 10^6$ N/cm^2; $\nu_2 = 0.13$, and $\rho_0/\rho = 1.7$. The plate is excited by a concentrated force with frequency 1 kHz and amplitude 1 kN.

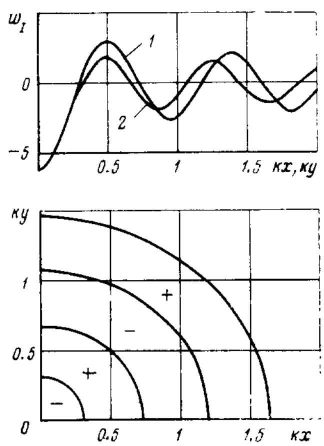

Fig.2

In Fig.2 may be found terms showing the variation of the imaginary part of the deflection w_I along the orthotropic axes x and y (curve 1 and 2) which, as in the isotropic case, yields the principal contribution to the deflection value in a neighborhood of the point of application of the force. Here may also be found the nodal lines w_I (because of the symmetry of the solution, these curves are presented only in the first quadrant of the xy-plane). The form of the curve may be approximated by ellipses oriented in the direction of maximal rigidity and having the semi-axis $j_{0,s}/b_x$ (along x) and $j_{0,s}/b_y$ (along y), where $j_{0,s}\ (s = 1, 2, \ldots)$ are the zeroes of the Bessel function J_0, $b_y/b_x = (E_1/E_2)^{1/4}$. In the isotropic case ($E_1 = E_2 = E$), the nodal lines assume the form of concentrated circles with radii proportional to the roots $j_{0,s}$.

The method presented above for solving oscillation problems occurring with infinite plates in a liquid excited by concentrated forces may also be applied to construct solutions in the case of a plate subjected to a concentrated moment. From a previous study /5/, it follows that we may associate with the effect of a concentrated momentum the derivative of the δ-function in the same direction as the direction of the momentum. In this case, a solution of the system (1.1) is written in the form of the derivative along the corresponding direction of the solution obtained from the case of action of a concentrated force.

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